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# Multiplying negative numbers in different sets

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# Multiplying Negative Numbers in Different Sets

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**Abstract.** The article considers the multiplication of negative numbers, says about the ability to take out minus one and work with negative numbers as with positive ones, while counting orders of minus one. If «minus times minus will be plus» then when a negative number is raised to a power a positive number is obtained which is uncharacteristic for mathematics as for the structure of numbers. If «minus times minus will be minus» then computational errors occur. Alternating powers of negative numbers are the basis of alternating series, their conditional and absolute convergence. We can consider two sets of positive and negative numbers and say that if we work on an adjacent set of both positive and negative numbers then «minus times minus will be plus», and if we work only on the set of negative numbers then «minus times minus will be minus». Then when raising negative numbers to a power negative numbers will be obtained, as it should be. And when multiplying two negative numbers, a negative number will be obtained if we work on a set of negative numbers. Then we can plot exponential and logarithmic functions with negative bases.



## INTRODUCTION

One of the cornerstones of working with negative numbers is the statement that when two negative numbers are multiplied, a positive number is obtained. When raising negative numbers to a power, the signs of the resulting numbers alternate – at even power we get a positive number, at odd power –a negative one.

The objectives of this study are to:

- study contradictions when multiplying negative numbers;
- solve examples with different laws of multiplication of negative numbers;
- evaluate multiplication of negative numbers on different sets;
- construct exponential and logarithmic functions on negative bases.

## DISCUSSION

Let's review an example: -2-(-2)=-2+2=0. Here minus by minus when subtracting will be plus, which can be confirmed by reasoning - if you subtract the number from the numberitself then you get zero.But is it possible to apply this rule that «minus by minus will be plus» to multiplication? Can (-2)*(-1) be checked by reasoning? (-2)*1=(-2) - if the number is taken once then the number itself will be obtained. Butwhat happens if the number is taken minus one time? Thus we conclude that «minus by minus will be plus» from the addition, and apply it to multiplication.

### Minus Times Minus

Multiplication can always be checked by addition:

$$3\cdot 6=6+6+6=18;$$
$$(-3)\cdot 6=(-3)+(-3)+(-3)+(-3)+(-3)+(-3)=-18;$$
$$3\cdot(-6)=(-6)+(-6)+(-6)=-18;$$

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$(-3)\cdot(-6)$ – we can't check by addition…

We can check multiplication by a positive number by adding: $3*(-3)=(-3)+(-3)+(-3)=-9$. But $(-4)\cdot(-4)=(-1)^2\cdot 4^2=(-1)^2\cdot 16$. Thus, when we multiply negative numbers, we can take out minus one: $(-3)*(-6)=(-1)*3*(-1)*6=(-1)^2*18$.

If during mathematical operations with negative numbers we take out a minus one, then they look like this:

$$(-4)/(-4)=((-1)/(-1))*(4/4)=1;$$
$$-4-4=(-1)*4+(-1)*4=(-1)*(4+4)=(-1)*8=-8;$$
$$-4-(-4)=(-1)*4-(-1)*4=(-1)*(4-4)=(-1)*0=0.$$

If when working with negative numbers we take out minus one, then we work with negative numbers as with positive, and minus one accumulates: $(-4)^2=(-4)\cdot(-4)=(-1)^2\cdot 16$. But we can always take out minus one:

$$(-3)*6=(-1)*3*6=(-1)^1*18;$$
$$3*(-6)=3*(-1)*6=(-1)^1*18;$$
$$(-3)*(-6)=(-1)^2*18.$$

Minus one can always be taken out during multiplication and we can work with negative numbers as with positive ones.

$$(-3)*(-6)*(-7)=(-1)^3*3*6*7=(-1)^3*126;$$
$$(-4)*(-5)*(-8)*(-9)=(-1)^4*4*5*8*9=(-1)^4*1440;$$
$$(-4)*(-5)+(-8)*(-9)= (-1)^2*4*5+(-1)^2*8*9=(-1)^2*20+(-1)^2*72=(-1)^2*92;$$
$$(-4)/(-5)+(-8)*(-9)=0{,}8+(-1)^2*72;$$

To avoid non-obvious errors when multiplying negative numbers with each other, you can always take out minus one and accumulate its powers.

When working with negative numbers, it is always confusing to alternate the signs of the powers of negative numbers. If we compare negative numbers with «debt» and positive numbers with «profit» then with multiple «multiplication»,«cloning» of debt we make a profit. It is easier to work with numbers having some kind of applied reinforcement. With multiple multiplication of negative numbers, «minus» should accumulate, «debt» should increase, the negative number should increase and not become a «plus» in any way. It is the alternation of the signs of the powers of negative numbers that is wrong in terms of logic.

$$(-3)^1=-3;$$
$$(-3)^2=(-3)*(-3)=9;$$
$$(-3)^3=(-3)*(-3)*(-3)=-27;$$
$$(-3)^4=(-3)*(-3)*(-3)*(-3)=81;$$
$$(-3)^5=(-3)*(-3)*(-3)*(-3)*(-3)=-243\ldots$$

When cloning negative numbers, we get positive ones half the time with even powers.

Exponentiation is a multiple multiplication. With repeated multiplication of a negative number the «debt» should increase, but in our case with an even number of factors, suddenly the debt becomes «profit», and with an odd number of factorsit becomes «debt» again. It is the alternation of signs when exponentiating, multiplying negative numbers repeatedly that shows the incorrectness of the statement that «a minus by a minus will be a plus».

If we take out minus one, then the powers of minus three look like this:

$$(-3)^1=(-1)^1*3;$$
$$(-3)^2=(-3)*(-3)=(-1)^2*9;$$
$$(-3)^3=(-3)*(-3)*(-3)=(-1)^3*27;$$
$$(-3)^4=(-3)*(-3)*(-3)*(-3)=(-1)^4*81;$$
$$(-3)^5=(-3)*(-3)*(-3)*(-3)*(-3)=(-1)^5*243\ldots$$

There is a «minus» - «debt», but there is a «plus» - «profit». When we accumulate debt, we make a profit, when we multiply the negative number, we get a positive one.

If we continue these arguments, then when we multiply a negative number, we should get a negative one so «minus by minus will be minus». Then the exponentiation of a negative number is as follows:

$$(-3)^1=-3;$$
$$(-3)^2=(-3)*(-3)=-9;$$
$$(-3)^3=(-3)*(-3)*(-3)=-27;$$
$$(-3)^4=(-3)*(-3)*(-3)*(-3)=-81;$$
$$(-3)^5=(-3)*(-3)*(-3)*(-3)*(-3)=-243\ldots$$

It follows from exponentiation that when negative numbers are multiplied by each other, negative numbers are also obtained. Let's go through examples of calculations if «minus by minus will be minus»:

$$(-3)*(-6)*(-7)=(-18)*(-7)=-126;$$
$$(-4)*(-5)*(-8)*(-9)=(-20)*(-72)=-1440;$$

$$(-4)*(-5)+(-8)*(-9)=(-20)+(-72)=-92;$$
$$(-4)/(-5)+(-8)*(-9)=(-0.8)+(-72)=-72.8;$$
$$(-3)*6=3*(-6)=(-3)*(-6)=-18.$$

The division of negative numbers can be replaced by multiplication therefore, when dividing negative numbers by each other, a negative number is also obtained: (-2)/(-2)=((-1)/(-1))*(2/2)=(-1)*1=-1. Then $(-1)^1=(-1)^2=(-1)^3=(-1)^4\ldots=-1$.

Thus when multiplying negative numbers, we can take out minus one, count its orders and work with negative numbers as with positive ones. Also if applied categories are applied to negative numbers, to take negative numbers as a debt and positive ones as a profit, then when «cloning» debt, debt should also be obtained, when raising negative numbers to a degree, negative numbers should also be obtained.If we continue to speculate on multiplying negative numbers, then when multiplying negative numbers, a negative number should also be obtained, so «minus by minus will be minus».

## Set Theory

When multiplying negative numbers it is generally accepted that «minus by minus will be a plus», but one can also consider the possibility that «minus by minus will be a minus».In the first case it is illogical to alternate the powers of negative numbers, and in the second case computational contradictions occur, for example, (-3)/(-3)=-1 – a number when divided by itself gives minus one, not one, but in this case a negative number when divided by itself gives minus one, one in the set of negative numbers.

If we assume that «minus by minus will be a plus» then the signs of the powers of negative numbers alternate, which is uncharacteristic for mathematics as for a system of numbers. In higher mathematics conditional convergence, alternating series, their absolute and conditional convergence are based on the alternation of signs of degrees of minus one.

When positive numbers are multiplied by one in the set of positive numbers, the numbers remain unchanged. When negative numbers are multiplied by minus one, the numbers again remain the same. One and minus one do not change positive and negative numbers respectively. Thus, the sets of positive and negative numbers are symmetric and opposite. Minus one is the unit of the set of negative numbers. a*1=a, where one is a unit on different sets of numbers, for positive numbers it is 1 and for negative numbers it is (-1).

$(-3)^0=-1$, a negative number to zero power gives the unit of the negative set - minus one. So we separate the sets of positive and negative numbers. Symmetrically reflect a positive set and get a negative one. Then (-3)/(-3)=(-1) can be explained that a number when divided by itself gives one in its set. For positive numbers it is 1: 3/3=1, and for negative it's (-1): (-3)/(-3)=(-1). Then we matched 3~(-3) and 1~(-1). We assign three with minus three, and one respectivelywith minus one. From division (-3)/(-3)=(-1) one can go to multiplication (-3)*(-1)=(-3). So minus one on the set of negative numbers does not change the original number as one on the set of positive numbers.

(-3)*1=(-3) and (-3)*(-1)=(-3), then (-3)*±1=(-3). But +1 refers to a set of positive numbers, and (-1) to a set of negative numbers, the same number when multiplied by different numbers gives the same answer. But these results are obtained in different sets: (-3)*1=(-3) - in adjacent set, (-3)*(-1)=(-3) - in negative set. We work with numbers in different sets and go from set to set. We work in different sets and according to different laws.

## Transitions Theory

Let's examine the calculations assuming that «a minus by a minus will be a minus».

(-2)*(3-3)=(-6)-6=-12 which is not correct. But if we distinguish minuses: minus before a number as a sign of a number, and minus as a subtraction of numbers, then: (-2)*(3 – 3)=(-2)*3 – (-2)*3=(-6) – (-6)=0, which is correct.

For clarity of reasoning these minuses can still be distinguished. Subtraction can also be separated by spaces. Negative numbers will be written in brackets for ease of understanding. The number minus itself gives zero. (3 – 3)*(-2)=3*(-2) – 3*(-2)=(-6) – (-6)=0 – because there is no multiplication of minus by minus, there is no contradiction. Examine another example: ((-3) – 3)*(-2)=(-3)*(-2) – 3*(-2) – what will be the result of this expression?

If minus times minus equals plus, then 6 – (-6)=12 – correct.

If minus times minus equals minus, then (-6) – (-6)=0 – incorrect.

(3 – (-3))*(-2)=3*(-2) – (-3)*(-2).

If minus times minus equals plus, then (-6) – 6=-12 – correct.

If minus times minus equals minus, then (-6) – (-6)=0 – incorrect.

Let's define operations with subtraction and negative numbers. What is equal to 3 − (-3)? Examine five cases:

(-3)+3=0;
(-3) − 3=(-6);
(-3)+(-3)=(-6);
(-3) − (-3)=0;
3 − (-3)=6;
(-3)*((-2)+3)=(-3);

(-3)*((-2)+3)=(-3)*(-2)+(-3)*3=(-6)+(-9)=(-15); which is not correct because we are working in different sets of numbers. But if only in the set of negative ones: (-3)*((-2)+(-3))=(-3)*(-2)+(-3)*(-3)=(-6)+(-9)=-15. We go from set to set and get wrong values.

Maybe in the set of negative numbers work according to their own laws where minus by minus will be minus, and on the adjacent set where both positive and negative numbers are present - minus by minus will be plus[1]. On different sets we work according to different laws. So if we work on a set of negative numbers, then minus by minus will be minus. And if on a set of negative and positive numbers, we go through zero then minus by minus will be a plus. We seem to be working on the edge of sets where completely different laws work. On the set of positive numbers respectively plus by plus will be plus.

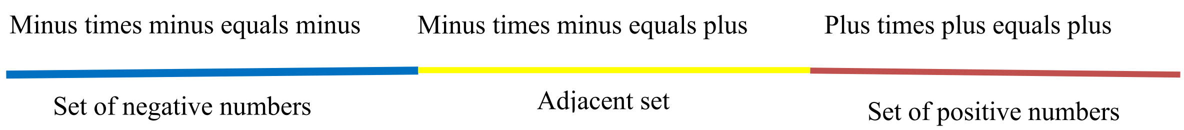


**FIGURE** 1. Multiplication signs for different sets.

Then (-3)*((-3)+2)=(-3)*(-1)=3=(-3)*(-3)+(-3)*2=9+(-6)=3 which is correct.

(-3)*((-2)+3)=(-3)*1=(-3)=(-3)*(-2)+(-3)*3=6+(-9)=(-3); which is also correct. In these examples we have transitions through zero, transitions from positive numbers to negative ones, so we work according to the rule «minus times minus is plus».

(-3)*((-2)+(-4))=(-3)*(-6)=(-18)=(-3)*(-2)+(-3)*(-4)=(-6)+(-12)=(-18), only negative numbers are present here, and we work according to the rule «minus times minus is minus».

Then there will be no alternation of signs of powers of negative numbers, and with negative numbers we will work in negative categories where minus by minus will be minus.

(-4)*((-2)+(-3))=(-4)*(-5)=(-4)*(-2)+(-4)*(-3)=(-8)+(-12)=(-20) – there is no transition through zero, we work in the set of negative numbers, so we use the rule«minus by minus will be minus».

(-4)*((-2)+3)=(-4)*(1)=(-4)=(-4)*(-2)+(-4)*3=8+(-12)=(-4) – there is a zero crossing here, so we work in the adjacent set according to the rule «minus times minus is plus».

In the first case minus times minus will be minus. And in the second case minus by minus will be a plus, as we work with a positive number 3 and get into an adjacent set.

If the examples for calculation are long, but they also contain zero crossings, then we work according to the rule «minus times minus is plus».

(-4)*((-2)+(-3))+(-4)*((-2)+3)=(-4)*(-5)+(-4)*1=20+(-4)=16=(-4)*((-2)+(-3)+(-2)+3)=(-4)*(-4)=16.

Perhaps for long examples both rules can work. It is better to raise negative numbers to a power according to the rule «minus times minus is minus»:

$(-2)^4$+(-4)*((-2)+(-3))=(-16)+(-4)*(-5)=(-16)+(-20)=(-36);
$(-2)^4$+(-4)*((-2)+3))=(-16)+(-4)*1=(-16)+(-4)=(-20);
$(-2)^4$+(-4)*((-3)+2)=(-16)+(-4)*(-1)=(-16)+4=(-12);

Then we calculate the left part of the last example according to the rule «minus times minus will be minus», and the right part – according to the rule «minus times minus will be plus». But we raise negative numbers to a power better according to the rule: «a minus by a minus will be a minus»: $(-2)^4$*(-4)*((-3)+2)=(-16)*(-4)*(-1)=(-16)*4=(-64).

In the last example we raise (-2) to an even degree according to the rule «minus by minus will be minus» and then we work according to the rule «minus by minus will be plus» because we have a transition through zero, we work in an adjacent set.

Then according to this rule if earlier (-3)*(-4)=12, now (-3)*(-4)=(-12) - only for negative numbers.

Thus, it is possible to solve the contradiction when multiplying negative numbers and working with them. If we fall into an adjacent set, in calculations there are both positive and negative numbers, then we work according to the rule «minus by minus will be plus», and if only in the set of negative numbers- then according to the rule «minus by minus will be minus».

## Negative Number Theory

If we accept a set of negative numbers, on which its own laws apply, on which a minus times minus will be a minus for a set of negative numbers, then we can count expressions like this: $\sqrt{-25} = -5и\sqrt{-36} = -6$etc. But since mathematics works on one common set of numbers, calculations are possible in numerical expressions like this: $\sqrt{-25} + \sqrt{-36} = -5 - 6 = -11$. Then there is no need for complex numbers in principle and i, because (-1)*(-1)=(-1). As $(i)^2=(-1)$[2], then according to the theory of negative numbers i=-1. According to the theory of complex numbers [3]: $\sqrt{-25} = 5i$, $(5i)^2=(-25)$; and$\sqrt{-36} = 6i$; $(6i)^2=(-36)$ etc.[4]

You can apply different multiplication laws on different sets. In one example different laws can be applied.

$(-2)^4+(-2)*((-2)+4)=(-16)+(-2)*2=(-16)+(-4)=(-20)\neq(-2)*((-2)^3+(-2)+4)=(-2)*((-8)+2)=(-2)*(-6)=(-12)$;

$(-2)^4+(-2)*((-3)+4)=(-16)+(-2)*1=(-16)+(-2)=(-18)\neq(-2)*((-2)^3+(-3)+4)=(-2)*((-8)+1)=(-2)*(-7)=(-14)$;

Then we must accept that on an adjacent set, when raising negative numbers to an even power, a positive number is obtained, «a minus times a minus will be a plus».

$(-2)^4+(-2)*((-2)+4)=16+(-2)*2=16+(-4)=12=(-2)*((-2)^3+(-2)+4)=(-2)*((-8)+2)=(-2)*(-6)=12$, which is correct.

$(-2)^4+(-2)*((-3)+4)=16+(-2)*1=16-2=14=(-2)*((-2)^3+(-3)+4)=(-2)*((-8)+1)=(-2)*(-7)=14$;

$(-2)*((-4)+3)=(-2)*(-1)=2=(-2)*(-4)+(-2)*3=8+(-6)=2$;

$(-2)*((-3)+4)=(-2)*1=(-2)=(-2)*(-3)+(-2)*4=6+(-8)=(-2)$;

If you solve from left to right, as in the source code, then there are no such problems. Then, when multiplying any two negative numbers, we work according to the rule «minus by minus will be minus».

$(-2)^4+(-2)*(-4)+3= (-16) +(-2)*(-4)+3=(-18)* (-4)+3=(-72)+3=(-69)$;

(-2)f4a(-2)c(-4)a3=(-16)a(-2)c(-4)a3=(-18)c(-4)a3=(-72)a3=(-69);

$(-2)^4+(-2)*(-4)+2=(-16)+(-2)*(-4)+2=(-18)*(-4)+2=(-72)+2=(-70)$;

(-2)f4a(-2)c(-4)a2=(-16)a(-2)c(-4)a2=(-18)c(-4)a2=(-72)a2=(-70).

In the source code operations are performed strictly from left to right, one operation at a time, there are no brackets, different sequences of actions[5].And therefore, only the rule «a minus by a minus will be a minus» always works here, when multiplying two negative numbers, a negative number is obtained, as in the source code we consistently work with only one operation. We step from operation to operation: (any negative number)*(any negative number)=(some negative number). Usually the source code gives answers to any questions.Maybe for our complex mathematics it is not worth looking for easy answers.

We can enter the reverse as a mathematical operation. Then the number 123 at the reverse will correspond to the number 321. There are numbers that, when reversed, correspond to themselves, these are symmetric numbers. For example, 111 or 2002, or 22,022,022. In this case the number is treated as a symbol.

## Graphs of Functions with Negative Bases

If we accept a set of negative numbers, which has its own laws, its own rule for multiplying negative numbers, then on this «negative» set we can build graphs of negative functions [6]. Thus, it is possible to separate the set of positive and negative numbers. And work in each set separately and similarly. Then we can plot exponential and logarithmic functions with negative bases and get smooth functions. Let's give examples of graphs of functions $y=(-2)^x$and $y=\log_{(-2)}(x)$ andalso $y=(-0,5)^x$and $y=\log_{(-0,5)}(x)$. Graphs of the function $(-2)^x$and $2^x$ will be symmetrical about the OX axis.

If we assume that minus by minus will be minus, then the graph of the function $y=a^x$, where a<0, will be located in a negative half-plane and symmetrical to the graph $y=a^x$, where a>0 abouttheOXaxis [7]. Then $(-2)^0=-1$. Then uncertainty $0^0$->1 or $0^0=1$, this can be verified by raising a number close to zero to a degree close to zero. Zero, like many other numbers, to the degree tending to zero gives one.

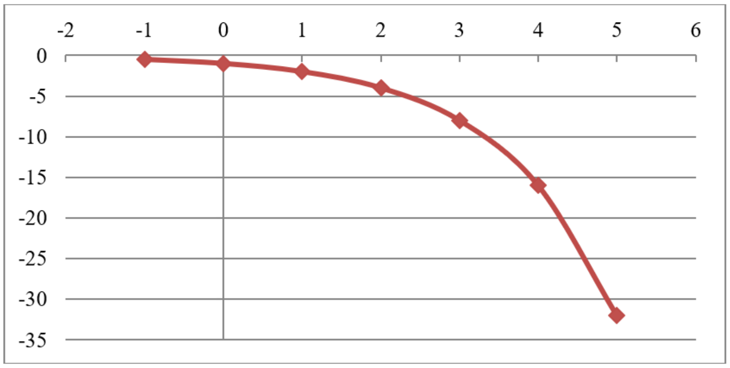


**FIGURE 2.** Function graph $y=(-2)^x$.

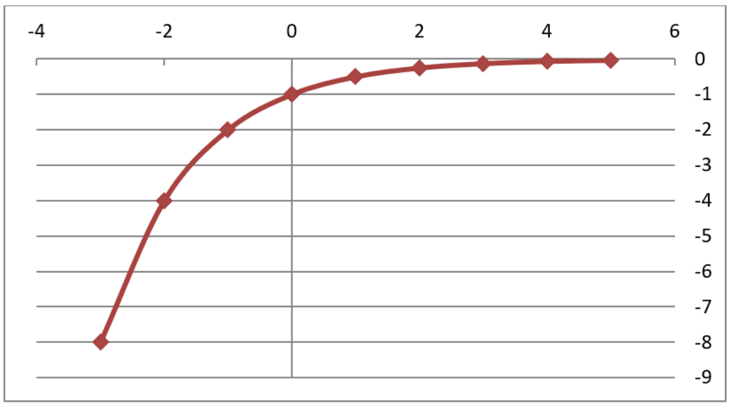


**FIGURE 3.** Function graph $y=(-0,5)^x$.

The graphs of the functions $(-0.5)^x$and $0.5^x$ will be symmetrical about the OX axis. Function $y=(-2)^x$is decreasing. Functiony$=(-0,5)^x$ is increasing. Functions $y=(-2)^x$and $y=(-0,5)^x$are symmetrical about the OY axis. At the same time the graphs $y=(-2)^x$are $y=\log_{(-2)}(x)$ are symmetrical about the liney=x – these are inverse functions. But inverse functions are not inverse numbers and their product is not equal to 1 [8].

For example, $y=\log_2 x$ and $y=2^x$ are symmetrical about the line y=x and they are inverse functions. Inverse functions are obtained by interchanging x and y in the original function [9]. It is obvious that$\log_2 x*2^x\neq 1$.

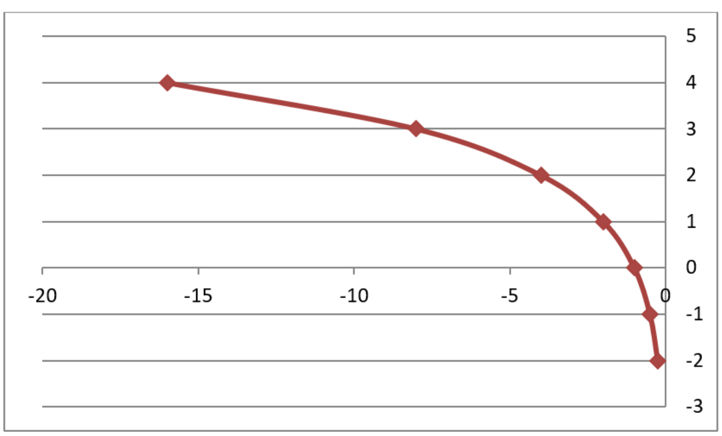


**FIGURE 4.** Function graph $y=\log_{(-2)}(x)$.

The function graphs $\log_{(-2)}(x)$and$\log_2(x)$will be symmetrical about the OY axis.The function $y=\log_{(-2)}(x)$is decreasing. The function $y=\log_{(-0,5)}(x)$is increasing.

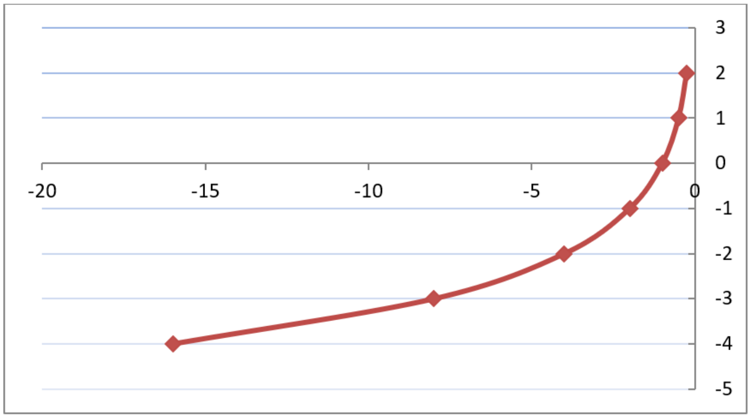


**FIGURE 5.** Function graph $y=\log_{(-0,5)}(x)$.

The function graphs $\log_{(0,5)}(x)$andlog$_{(-0,5)}(x)$will be symmetrical about the OY axis. At the same time, the graphs $y=(-0,5)^x$and$y=\log_{(-0,5)}(x)$ are symmetrical about the straight line y=x, these are inverse functions. $(-0,5)^{-1}=1/(-0,5)=-2$. A number that is raised to a power is perceived as a symbol.

## CONCLUSION

Different laws of multiplication of negative numbers will apply on different sets. If we work on an adjacent set, where both positive and negative numbers are present, then we multiply negative numbers according to the rule «minus by minus will be plus» and if on a set with only negative numbers, then «minus by minus will be minus». If we work with the source code, then multiplying two negative numbers will also result in a negative number. On the set of negative numbers it is possible to construct logarithmic and exponential functions on negative bases, while considering that when raising negative numbers to even degrees, a negative number will also be obtained. Plots of the exponential and logarithmic functions at one base will be inverse functions and symmetric about the line y=x.

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